University of Pune

Three Year B. A. Degree Program

Syllabus for F.Y.B.A. Mathematical Statistics

(With effect from Academic Year 2013-2014)

Submitted by

Prof. S. R. Deshmukh
Chairperson,
Board of Studies in Statistics

1) Title of the program: Three Year B. A. Degree

2) Preamble to the syllabus: The word ***Statistics*** is used in different ways in different contexts. To a cricket fan, Statistics is the information about runs scored or wickets taken by a player. To the manager of a manufacturing unit, Statistics may be the information about the process control. To a medical researcher investigating the effects of a new drug, Statistics are evidence of research efforts. To a college student, Statistics are the grades or marks scored in a course. Thus, in all these illustrations Statistics word refers to quantitative data in the area under study. Statistics as a subject is an important branch of knowledge and is devoted to various techniques of collection, presentation, analysis and interpretation of data. It is a science of learning from data.

Statistics provides tools for making decisions when conditions of uncertainty prevail. Hence these tools and techniques are used in almost all fields. Statistics is indispensable for people working in fields like agriculture, business, management, economics, finance, insurance, education, biotechnology and medical science etc. Since last two decade, with the help of computers large amount of data can be handled and more sophisticated statistical techniques can be used in an effective manner. Knowledge of different aspects of Statistics has become crucial. There is a continuous demand for statisticians in every field – education, industry, software and research. The syllabus of the three Year B. Sc. degree course in Statistics is framed in such a way that the students at the end of the course can apply judiciously the statistical tools to a variety of data sets to arrive at some conclusions.

Statistics can be divided into two broad categories, (1) exploratory statistics or descriptive statistics, which is concerned with summarizing data and describing these data, and (2) confirmatory statistics or inferential statistics, which is concerned with making decisions about the population based on the sample.

Up to higher secondary school, students are mostly exposed to descriptive statistics. At the first year a student can take any one of the four subjects related statistics, such as Statistics, Applied Statistics, Mathematical Statistics and Statistical Prerequisites. If the student continues with these subjects at the second year and third year, it is expected that at the end of the degree course a student is able to apply the statistical tools to real life data.

3) Introduction: Three Year B. A. degree program is of three years duration, with semester pattern for the second and third year and annual examination pattern for the first year.

The structure of **Bachelor of Arts (B. A.) is as follows.** The student joining the First Year B.A. Course has to take six subjects from 13 groups. The student cannot take more than one subject from one group. There are four subjects related to statistics. These are Statistics (Group L), Applied Statistics (Group L), Mathematical Statistics (Group J) and Statistical Prerequisites (Group K).

4) Eligibility

First Year B.A.

Higher secondary school certificate examination of the Maharashtra State Board of Higher Secondary Education or an equivalent examination of any other statutory Board or University with English as a passing subject.

Detailed Syllabus for F.Y.B.A. Mathematical Statistics

Discrete Probability and Probability Distributions

Objectives: The main objective of this course is to introduce to the students the basic concepts of probability, axiomatic theory of probability, concept of random variable, probability distribution (univariate and bivariate) discrete random variables, expectation and moments of probability distribution. By the end of the course students are expected to be able

(i) to distinguish between random and non-random experiments.
(ii) to find the probabilities of events.
(iii) to obtain a probability distribution of random variable (one or two dimensional) in the given situation, and
(iv) to apply standard discrete probability distribution to different situations.

1. Review of probability, conditional probability, independence (8)

1.1 Experiments/Models, Ideas of deterministic and non-deterministic models. Random Experiment, concept of statistical regularity.
1.2 Definitions of - (i) Sample space, (ii) Discrete sample space: finite and countably infinite, (iii) Event, (iv) Elementary event, (v) Complement of an event. (vi) Certain event (vii) Impossible event
1.3 Concept of occurrence of an event.
1.4 Algebra of events and its representation in set theory notation.
1.5 Occurrence of following events.
  (i) at least one of the given events,
  (ii) none of the given events,
  (iii) all of the given events,
  (iv) mutually exclusive events,
  (v) mutually exhaustive events,
  (vi) exactly one event out of the given events.
1.6 Classical definition of probability and its limitations.
1.7 Probability model, probability of an event, equiprobable and non-equiprobable sample space,
1.8 Axiomatic definition of probability.
1.9 Definition of conditional probability of an event.
1.10 Definition of independence of two events $P(A \cap B) = P(A) \cdot P(B)$
1.11 Pairwise independence and mutual independence for three events
1.12 Multiplication theorem $P(A \cap B) = P(A) \cdot P(B|A)$.
  Generalization to $P(A \cap B \cap C)$.

2. Bayes' Theorem (4)

2.1 Partition of the sample space

2.2 Proof of Bayes' theorem. Applications of Bayes' theorem in real life

3. Univariate Probability Distributions (Defined on Discrete Sample Space) (6)

3.1 Concept and definition of a discrete random variable.

3.2 Probability mass function (p.m.f.) and cumulative distribution function (c.d.f.), F(·) of discrete random variable, properties of c.d.f..

3.3 Mode and median of a univariate discrete probability distribution.

4. Mathematical Expectation (Univariate Random Variable) (10)

4.1 Definition of expectation (Mean) of a random variable, expectation of a function of a random variable, m.g.f. and c.g.f. Properties of m.g.f and c.g.f.

4.2 Definitions of variance, standard deviation (s.d.) and Coefficient of variation (c.v.) of univariate probability distribution, effect of change of origin and scale on mean, variance and s.d.

4.3 Definition of raw, central and factorial raw moments of univariate probability Distributions and their interrelations (without proof).

4.4 Coefficients of skewness and kurtosis based on moments.

5. Some Standard Discrete Probability Distributions - I (20)

5.1 Degenerate distribution (one point distribution), P(X=c) =1, mean and variance.

5.2 Uniform discrete distribution on integers 1 to n: p.m.f., c.d.f., mean, variance, real life situations, comments on mode and median.

5.3 Bernoulli Distribution: p.m.f., mean, variance.

5.4 Binomial Distribution: p.m.f.

$$p(x) = \binom{n}{x} p^x q^{n-x}; x = 0, 1, \dots, n, 0 < p < 1, q = 1 - p$$
$$= 0 \; otherwise$$

Notation : X ~ B(n, p).

Recurrence relation for successive probabilities, computation of probabilities of different events, mode of the distribution, mean, variance, m.g.f. and c.g.f. moments, skewness (comments when p = 0.5, p > 0.5, p < 0.5). Situations where this distribution is applicable.

5.5 Hypergeometric Distribution : p.m.f.,

$$p(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} \; x = 0, 1, \dots, \min\{M, n\} \;,$$
$$= 0 \; otherwise$$

Notation : X ~ H(N,M, n).

Computation of probability, situations where this distribution is applicable, binomial approximation to hypergeometric probabilities, mean and variance of the distribution.

End of the First Term

6. Some Standard Discrete Probability Distributions - II (20)

6.1 Poisson distribution: Notation : X ~ P(m).

$$p(x) = \frac{e^{-m}m^x}{x!}, x = 0,1,2, \dots, m > 0$$

m.g.f. and c.g.f. Moments, mean, variance, skewness and kurtosis. Situations where this distribution is applicable.

6.2 Geometric distribution: Notation: X ~ G(p),
Geometric distribution on support (0, 1, 2, ...,) with p.m.f. $p(x) = p\,q^x$.
Geometric distribution on support (1, 2, ...) with p.m.f. $p(x) = p\,q^{x-1}$.
$0 < p < 1$, $q = 1 - p$.
Mean, variance, m.g.f. and c.g.f. Situations where this distribution is applicable.

7. Bivariate Discrete Probability Distribution (10)

7.1 Definition of two-dimensional discrete random variable, its joint p.m.f. and its distribution function and their properties, concept of identically distributed r.vs.
7.2 Computation of probabilities of events in bivariate probability distribution.
7.3 Concepts of marginal and conditional probability distributions.
7.4 Independence of two discrete random variables based on joint and marginal p.m.f.s

8. Mathematical Expectation (Bivariate Random Variable) (18)

8.1 Definition of raw and central moments, m.g.f , c.g.f.
8.2 Theorems on expectations of sum and product of two jointly distributed random variables.
8.3 Conditional expectation.
8.4 Definitions of conditional mean and conditional variance.
8.5 Definition of covariance, coefficient of correlation, independence and uncorrelatedness of two variables.
8.6 Variance of linear combination of variables Var(aX + bY).
8.7 Additive property for binomial and Poisson distributions.
8.8 Introduction of negative binomial distribution as sum of k i.i.d. geometric random variables. Statement of p.m.f., mean and variance.
8.9 Conditional distribution of X given (X+Y) for binomial and Poisson distributions.

Recommended Books:

1. Agarwal B. L. (2003). Programmed Statistics, second edition, New Age International Publishers, New Delhi.
2. Gupta, S.C. and Kapoor, V. K. (1983). Fundamentals of Mathematical Statistics, Eighth Edition, Sultan Chand and Sons Publishers, New Delhi.
3. Hoel P. G. (1971). Introduction to Mathematical Statistics, John Wiley and Sons, New York.
4. Hogg, R. V. and Craig R. G. (1989). Introduction to Mathematical Statistics, ed. 4. MacMillan Publishing Co., New York.

5. Mayer, P. (1972). Introductory Probability and Statistical Applications, Addison Wesley Publishing Co., London.
6. Mood, A. M. and Graybill, F. A. and Boes D.C. (1974). Introduction to The Theory of Statistics, Ed. 3, McGraw Hill Book Company.
7. Ross S. (2002). A First Course in Probability, Sixth Edition, Pearson Education, Inc. & Dorling Kindersley Publishing, Inc.

Reference Websites

1. www.stats.unipune.ac.in(100 Data sets for Statistics Education by Dr. Anil P. Gore, Dr. Mrs. S. A. Paranjpe and Madhav B. Kulkarni available in ISPS folder).
2. http://www.freestatistics.tk (National Statistical Agencies)
3. www.psychstat.smsu.edu/sbk00.htm (Online book)
4. www.bmj.bmjournals.com/collections/statsbk/index.shtml
5. www.statweb.calpoly.edu/bchance/stat-stuff.html
6. www.amstat.org/publications/jse/jse-data-archive.html (International journal on teaching and learning of statistics)
7. www.amstat.org/publications/chance (Chance magazine)
8. www.statsci.org/datasets.html (Data sets)
9. www.math.uah.edu/stat (Virtual laboratories in Statistics)
10. www.amstat.org/publications/stats (STATS : the magazine for students of Statistics)
11. www.stat.ucla.edu/cases (Case studies in Statistics).
12. www.statsoft.com
13. www.statistics.com
14. www.indiastat.com
15. www.unstat.un.org
16. www.stat.stanford.edu
17. www.statpages.net
18. www.wto.org
19. www.censusindia.gov.in
20. www.mospi.nic.in
21. www.statisticsofindia.in